2024

Time :As in Programme

Full Marks : 100

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer all Questions. 1x10

a. If $\gcd(a,b)=d$, then $\gcd\left(\frac{a}{d},\frac{b}{d}\right)=$ ____

b. There are ____ no of prime numbers.

c. If $a\mid bc \Rightarrow a\mid c$. (True/False)

d. Let {0,2,4} be a subring of the ring Z_6, the integer Modulo 6. The unity of {0,2,4} is ____.

e. Is $Z\oplus Z$ is an integral domain ? (Yes/No)

f. A={1,w,w^2} is a group under multiplication. (True/false)

g. Every subgroup of cyclic group is abelian. (True/false)

h. Find the generators of Z_{10}.

i. a|1 if and only if a = ____.

j. The equation ax + by = c has a solution if and only if ____.

(Turn Over)

PART-II

2. Answer the following questions. 2x9

a. Show that $(ab)^{-1}=b^{-1}a^{-1}$

b. Test $G=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times2}, \begin{matrix} ad-bc=1 \\ a,b,c,d\in R \end{matrix}\right\}$ is a group under matrix multiplication or not.

c. Find a subgroup of S_3.

d. Define cyclic group with two example.

e. Let $a\in$ a ring R, prove that $0a=a0=0$.

f. Is Z_6 a subring of Z_{12} ? Explain.

g. Prove that square of any integer is either of the form 3k or 3k + 1.

h. If k>0, prove that gcd(ka, kb)=k gcd(a,b).

i. Use Euclidean algorithm to find x & y $\in$ z S.t. gcd(56, 72)=56x+72y.

PART-III

3. Answer any eight questions of the followings. 5x8

a. If G is a group, then show that

i. The identity element of G is unique

ii. Every $a\in$ G has a unique inverse in G.

b. If H is a non-empty finity subset of a group G and H is closed under multiplication then H is a subgroup of G.

c. If G is a group and a, $b\in$ G then $(ab)^{-1}=b^{-1}a^{-1}$.

d. If every element of G has its own inverse then show G is an abelian group.

e. Let a$\in$ a ring R. Let S=$\{x\in R:ax=0\}$. Show that S is a subring of R.

f. Prove that ax + by = c has a solution if and only if d|c, where d=gcd(a,b).

g. If p is a prime number & $p|a^n$, then prove that $p^n|a^n$.

h. State and prove Fermat's Little Theorem.

i. State and prove Fermat's Theorem.

j. Let f_n denote the nth Fermat number. Then prove that $f_n = f_{n-1}^2 - 2f_{n-1} + 2$, where $n \geq 1$.

PART-IV

Answer any four of the following questions. 8x4

4. i. Prove that if a, b $\in$ G then $(ab)^2=a^2b^2$ iff G is abelian 4

   ii. Prove that intersection of two subgroups of a group G is again a subgroup of G. 4

5. Define centre of a group. Prove that the centre of a group G is a subgroup of G.

6. State and prove division algorithm

7. State and prove Wilson's theorem.

8. Solve the linear congruence $17x \equiv 9 \pmod{276}$.