2024

Time :As in Programme

Full Marks : 100

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer the following Questions. 1x10

   a. $Y = ax^2 + bx + c$ is an equation of a ____.

   b. Dependent variables and derivatives of linear differential equation are of ____ degree.

   c. Volume is a function of Three variables. (True/False)

   d. Integrating factor of equation Mdx + Ndy = 0 is ____.

   e. Curl of a conservative field is ____.

   f. Delta function has same dimension as that of the reciprocal of its ____.

   g. Magnetic field (Induction) is an example of a ____ field.

   h. Green's Theorem in a plane is a special case of ____ Theorem.

   i. $\vec{\nabla} \times (\hat{e}.\vec{r})\hat{e} =$ ____, where $\hat{e}$ is an unit vector.

   j. If $\vec{\nabla} \times \vec{A} = 0$, then $\vec{A}$ is an ____ field.

PART-II

2. Answer the following questions. 2x9

a. Prove that $\vec{\nabla} \cdot r^n = n \cdot r^{n-2} \cdot \vec{r}$, where $\vec{r}$ is a position vector.

b. State Stock's Theorem and give its mathematical form.

c. Solve the equation $y \cdot dx - x \cdot dy = 0$

d. Write the procedure to plot the curve of y=f(x).

e. Explain partial derivatives with one application.

f. Prove that scalar productive is distributive.

g. Explain the physical significance of curl of a vector field.

h. Prove that $x \cdot \delta(x) = 0$

i. Evaluate $\iint_s r \sin\theta . dr . d\theta$.

PART-III

3. Answer any eight questions of the followings. 5x8

a. Explain Jacobian and represent Jacobian of (u, v, w) with respect to (x, y, z)

b. Prove that curl $r^n \cdot \vec{r} = 0$, where $\vec{r}$ is a position vector.

c. What is Dirac-Delte function and prove that $\delta(ax) = \frac{1}{|a|} \cdot \delta(x)$.

d. Explain Laplacian $(\vec{\nabla})$ in terms of Orthogonal curvilinear co-ordinates.

e. Explain the conditions under which a generalised curvilinear co-ordinate system is orthogonal.

f. Derive an expression for exact differential.

g. Derive an expression for partial differentiation of vectors.

h. Explain scalar and vector field with examples.

i. What is Wronskian ? Give its physical significance and calculate the Wronskian of the function $e^{(p+iq)x}$.

j. Study the existence of the solution of the initial value problem $y^1 = x^2 + y^2, y(0) = 0$.

PART-IV

Answer any four of the following questions. 8x4

4. Derive an expression for second order homogeneous differential equation with constant coefficients.

5. Derive an expression for constrained Maximization using Lagrange Multipliers.

6. Explain Divergence of a vector field and give its physical significance with examples.

7. Explain the surface integral of a vector function and evaluate $\int_1^2 \int_0^x \frac{dx \cdot dy}{x^2 + y^2}$.

8. Explain scalar Triple product with its various properties and physical significance.