2024

Time :As in Programme

Full Marks : 100

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer all Questions. 1x10

a. $Y = \sin h^3(2x)$, find $\frac{dy}{dx}$.

b. Find the asymptotes parallel to the co-ordinate axes of the curve $x^2y^2 - a^2(x^2+y^2)=0$.

c. Find the value of $\int_0^{x/2} \sin^7 x dx$.

d. Evaluate $\lim_{x\to\infty} \frac{x^n}{e^x}$.

e. Write the reflexion property of the parabola.

f. $\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx$ is possible for ____ substitution.

(Turn Over)

g. Volume of solid, when the area bounded by the curve $Y=\sqrt{\cos x}$ and the x-axis from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ is revolved about x-axis is ____.

h. The area of revolution about x-axis of function $f(x)\geq 0$ is ____.

i. If $\overline{F}(t)=2ti-5j+t^2k$, $\overline{G}(t)=(1-t)i+\frac{1}{t}k$ then $F(t)\cdot\overline{G}(t)$ is ____.

j. If $\overline{r}(t)=(a\cos t)i+(a\sin t)j+bt(k)$, then $\overline{r}(t)$ is ____ when $t=\frac{\pi}{2}$.

PART-II

2. Answer the following questions. 2x9

a. Find the interval in which the function $f(x)=x+2\sin x$ is (i) concave up (ii) concave down

b. Find the angle of rotation that will eliminate xy-term from the equation $2x^2+xy+2y^2+x-y=0$

c. Find the eccentricity and distance from the pole to the directirix y of $x=\dfrac{4}{z+3\cos\theta}$.

d. Find the angle of rotation that will eliminate xy - term from the equation $2x^2+xy+2y^2+x-y=0$.

e. Find the eccentricity and distance from the pole to the directirix of $r = \frac{4}{z + 3\cos\theta}$.

f. Find the natural domain of

$\vec{r}(t) = (\ln|t-1|)\hat{i} + e^t\hat{j} + \sqrt{t}\hat{k}$

g. Evaluate $\int_0^1 \left(t^2\hat{i} + e^t\hat{j} - 2\cos\pi t\hat{k}\right)dt$

h. Find the Unit normal vector N(t) of

$\vec{r}(t) = \ln 2t\hat{i} + t\hat{j} at \ t = e$

i. Find the length of the curve $y=x^2$ from (0,0) to (2,4).

PART-III

3. Answer any eight questions of the followings. 5x8

a. Trace the cardioid $r = a(1+\cos\theta)$.

b. Evaluate $\lim_{x\to 1}(2-x)\tan\left(\frac{\pi x}{2}\right)$ using L hospital's rule.

c. Evaluate $\int \sec^7 x dx$ using reduction formula.

d. Describe the graph of the equation

$x^2 + y^2 - 4x + 8y - 21 = 0$

e. Find the vector function f such that $f(0) = f'(0) = i$ and $f''(0) = 2i + k$ and $f'''(t) = \cos ti + \sin tj + \frac{t}{\pi}k$.

f. Prove that $\cosh^{-1} x = \ln\left(x+\sqrt{x^2-1}\right)$

g. $Y = e^{4x}\sin(3x+2)$ find the nth derivative Y_n.

h. Find the asymptotes of

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$$

i. Find the volume of the solid generated when the region enclosed by $y=\sqrt{x+1}$,, $y=\sqrt{2x}$ and $y=0$ is revolved about x-axis.

j. Prove that $\left[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2$

PART-IV

Answer any four of the following questions. 8x4

4. Derive the formula for the volume of a right pyramid whose altitude is h and whose base is a square with sides of length a.

5. Suppose a particle moves through 3-space so that its position vector at time 't' is $\vec{r}(t) = e^{-2t}\hat{j} + t\hat{k}$ find the vector tangential and normal components of acceleration at time t=1.

6. Find all the asymptotes of the curve $x^3+y^3=3axy$.

7. Evaluate $\int x^2(\ln x)^3 dx$ using reduction formula.

8. Use cylindrical shells to find the volume of the solid generated when the region R in the 1st quadrant enclosed between $y=x$ and $y=x^2$ is revolved about y-axis.