2024

Time :As in Programme

Full Marks : 100

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer all Questions. 1x10

a. The point of inflexion of $f(x)=x^3$ is ____.

b. $\int x \cdot e^{3\ln x} dx =$ ____.

c. For the function $f(x)=\dfrac{3x+5}{7-x}$, the equation of vertical asymptote is ____.

d. $\int_{-1}^{2} |x|\, dx =$ ____.

e. $\int e^{\log}(1+\tan^2 x)_{dx} =$ ____

f. The formula for the length of the curve $x=f(t)$, $y=\phi(t)$ from t=α to t= β, is ____.

g. The volume of the solid generated by revolving the part of the curve x=f(y) between y=a and y=b around y-axis is given by ____.

(Turn Over)

h. $\nabla \cdot \nabla \times F =$ ____.

i. The volume of the solid generated when the region is enclosed by the curve $y=x^3$, $x=1$, $y=0$ is revolved about y-axis is ____.

j. The Unit tangent vector of $\vec{r}(t)=t^2\hat{i}+t^3\hat{j}$ at $t=2$ is ____.

PART-II

2. Answer the following questions. 2x9

a. Find the n^{th} derivative of $\cos^2 x$.

b. Find the whole length of the curve $x=\sin t$, $y=\cos t$.

c. Write the formula to obtain the volume of the solid generated by revolving $r=(1+\cos\theta)$ about the initial line.

d. Differentiate $r \times \dot{r}$

e. The acceleration of the particle is given by $a=12\cos 2t\, i - 8\sin 2t\, j + 16t\, k$. Find the velocity v when v vanishes at $t=0$.

f. Evaluate $\lim_{x \to 0} \frac{x^2+\sin x^2}{x^2+x^3}$.

g. Differentiate $\sinh^{-1}(\tanh x)\ w.r.t.x$.

h. Find the asymptotes parallel to the co-ordinate axes for the curve $x^2y^2 - a^2(x^2+y^2)=0$

i. Evaluate $\int_0^1 \left(t^2\hat{i}+e^t\hat{j}-2\cos \pi t\hat{k}\right)dt$.

PART-III

3. Answer any eight questions of the followings. 5x8

a. Prove that $\cosh^{-1} x = \ln\left(x+\sqrt{x^2-1}\right), x \geq 1$.

b. Find the asymptotes of the Curve

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0.$$

c. Use reduction formula to evaluate $\int \tan^6 x dx$

d. Describe the graph of the equation

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$

e. Suppose that a particle moves through 3 dimensional space so that its position vector at time t is

$\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}$ find the scalar tangential and Normal components of acceleration at time t.

f. Find the exact arc length of the curve 24xy-y^4+48 from y=2 to y=4.

g. Find the area of the surface generated by revolving the portion of the curve y=x^2 between x=1 and x=2 about y-axis.

h. Identify an sketch the curve xy=1.

i. If $\vec{r}(t)$ is a differentiable vector - valued function in 2 - space or 3 - space and $\|\vec{r}(t)\|$ is constant for all 't' prove that $\vec{r}(t)$ and $\overline{r}(t)$ are orthogonal vectors.

j. Find $\overline{r}(t)$ given that $\vec{r}(t) = \langle 3, 2t \rangle$ and $\vec{r}(1) = \langle 2, 5 \rangle$.

PART-IV

Answer any four of the following questions. 8x4

4. If $y = (\sin^{-1} x)^2$, then use Leibniz's theorem to prove that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$$

5. Evaluate $\int_0^{\pi/2} \ln \sin x dx$.

6. The part of the parabola $y^2=4ax$ bounded by the latus rectum revolves about the tangent at the vertex. Find the area of the curved surface of revolution thus generated.

7. Find the asymptotes to the curve $x^3 + y^3 = 3x^2$.

8. Evaluate $\int \log(x^2 + x + 2) dx$.