2024

Time :As in Programme

Full Marks : 100

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer all the following Question. 1x10

   a. The condition for all real roots in a quadratic equation is that the discriminant $b^2=4ac$ must be ____.

   b. The product of the roots of the quadratic equation $ax^2+bx+c=0$ is given by ____.

   c. A set of vectors is said to be linearly independent if the only solution to the equation

   $\alpha_1 v_1 + \alpha_2 v_2 + ... + \alpha_n v_n = 0$ is when ____.

   d. A subspace of a vector space is a subset that is closed under ____ and ____.

   e. A transformation T is linear if for all vectors u, v in v and scalars c, it satisfies the properties T(u+v)=____.

   f. A system of linear equations has a unique solution if the determinant of the coefficient matrix is ____.

   g. If two rows (or columns) of a determinant are identical, then its determinant is ____.

   h. The trace of a square matrix is the sum of its ____ elements.

i. The minimal polynomial of a matrix divides the ____ polynomial.

j. The rank of a matrix is equal to the number of ____ in its row echelon form.

PART-II

2. Answer the following question 2x9

a. If the roots of $x^2-7x+12=0$ are α and β. Find $\alpha+\beta$ and $\alpha\beta$.

b. Determine if the following vectors are linearly independent (1, 2, 3, 4), (2, 4, 6, 8) and (1, 1, 1, 1).

c. Solving using cramer's rule $x+2y=5$ and $3x-y=4$.

d. Find the dimension and a basis for the subspace $w=\text{span}(1, 2, 3), (2, 3, 4), (3, 5, 7)$ in R^3.

e. Find A^{-1} for $A=\begin{bmatrix}2 & 1\\ 3 & 4\end{bmatrix}$

f. What is meant by synthetic division ?

g. Find the eigen values of $A=\begin{bmatrix}2 & 3\\ 3 & 2\end{bmatrix}$.

h. What does the Carley - Hamilton theorem state ?

i. What is the characteristics equation of a matrix ?

PART-III

3. Answer any eight question 5x8

a. Solve $x^3-6x+2=0$ using Cardano's method.

b. Define a vector space. Give examples.

c. Show that a matrix is the combination of symmetric and skew symmetric matrix ?

d. Discuss completely the solution in the case of consistent systems.

$x_1 - x_2 + 2x_3 + 3x_4 = 1$

$2x_1 + 2x_2 + 2x_4 = 1$

$4x_1 + x_2 - x_3 - x_4 = 1$

$x_1 + 2x_2 + 3x_3 = 1$

e. Find A^{-1} if $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$.

f. Use Cramer's rule to find the solution of the given system of linear equations.

$2x - 3y = 7$ and $x+4y=1$

g. Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

h. Determine the row rank of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.

i. Find the eigen values of $A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 4 & 0 \\ 3 & 5 & -3 \end{bmatrix}$.

j. Verify Carley-Hamilton theorem for the matrix

$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ verify Carley-Hamilton theorem and find A^{-1}.

PART-IV

Answer any four of the following. 8x4

4. Solve $x^3 - x^2 - 8x + 12 = 0$ given that has a double root.
5. Explain how to calculate inverse by three method.
6. Define determinants and its properties with examples.
7. State and prove rank-nullity theorem.
8. Describe the procedure to find which matrix is consistent and inconsistent solution.